9.1 SEQUENCES

GENERAL FORMULA $a_n = f(n)$

FIND THE GENERAL FORMULA FOR

$-4, 6, -8, 10, -12 \ldots$

$n=1 \quad 2 \quad 3 \quad 4 \quad 5$

$-2(2) \quad +2(3) \quad -2(4) \quad +2(5) \quad -2(6)$

$(-1)^1(2)(2), \quad (-1)^2(2)(3), \quad (-1)^3(2)(4), \quad (-1)^4(2)(5), \quad (-1)^5(2)(6)$

$n=1 \qquad 2 \qquad 3 \qquad 4 \qquad 5$

$$a_n = (-1)^n 2(n+1)$$

SANITY CHECK: $a_4 = (-1)^4 2(4+1) = 2(5) = 10$ ✓

FIND THE G.F. FOR

$$\frac{4}{3}, \quad \frac{8}{5}, \quad \frac{16}{7}, \quad \frac{32}{9}, \quad \frac{64}{11}$$

$n=$ 1 2 3 4 5

4 8 16 32 64 NUMERATORS

2^2 2^3 2^4 2^5 2^6

$a_n = 2^{n+1}$ FOR NUMERATORS

3 5 7 9 11 DENOMINATORS

$2(1)+1$ $2(2)+1$ $2(3)+1$ $2(4)+1$ $2(5)+1$

$b_n = 2(n)+1 = 2n+1$ FOR DENOMINATORS

$$C_n = \frac{2^{n+1}}{2n+1}$$

RECONSIDER $3, 5, 7, 9, 11$

$+2 \quad +2 \quad +2 \quad +2$

A RECURSIVE DEFINITION OF A SEQUENCE

GIVES THE VALUE OF a_n IN TERMS OF n AND PREVIOUS TERMS

PLUS SOME INITIAL VALUES/CONDITIONS

RECURRENCE RELATION $\rightarrow$ $a_n = a_{n-1} + 2, \quad (n \geq 2)$

$a_1 = 3$

$a_{n-1} \neq a_n - 1$

FIND THE FIRST 5 TERMS OF THE SEQUENCE DEFINED BY $a_{n+1} = 1 - 2a_n \ (n \geq 1), \ a_1 = 1$

EASY TO WRITE

$a_1 = 1$

$n+1=2$ IE. $n=1$: $a_2 = 1 - 2a_1 = 1 - 2(1) = -1$

$n=2$: $a_3 = 1 - 2a_2 = 1 - 2(-1) = 3$

$n=3$: $a_4 = 1 - 2a_3 = 1 - 2(3) = -5$

$n=4$: $a_5 = 1 - 2a_4 = 1 - 2(-5) = 11$

$1, -1, 3, -5, 11, \ldots$ HARD TO FIND GENERAL FORMULA

NOW FIND a_{100} HARD TO USE TO FIND

NEED $a_{99}, a_{98}, \cdots, a_1$ ARBITRARY TERMS

FIND THE FIRST 6 TERMS OF THE SEQUENCE DEFINED RECURSIVELY

BY $a_n = a_{n-1} + a_{n-2}$, $a_1 = 1, a_2 = 1$
$(n \geq 3)$

$a_1 = 1$

$a_2 = 1$

$n=3$: $a_3 = a_2 + a_1 = 1 + 1 = 2$

$n=4$: $a_4 = a_3 + a_2 = 2 + 1 = 3$

$n=5$: $a_5 = a_4 + a_3 = 3 + 2 = 5$

$n=6$: $a_6 = a_5 + a_4 = 5 + 3 = 8$

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

FIBONACCI SEQUENCE

CONSIDER THE SEQUENCE DEFINED BY $a_{n+1} = (n+1)a_n$, $a_1 = 1 = 1!$
$(n \geq 1)$

"FACTORIAL"

$a_n = n(n-1)(n-2)\cdots 1 = n!$

$n=1$: $a_2 = 2a_1 = 2(1) = 2 = 2 \cdot 1 = 2!$

$n=2$: $a_3 = 3a_2 = 3(2) = 6 = 3 \cdot 2 \cdot 1 = 3!$

$n=3$: $a_4 = 4a_3 = 4(6) = 24 = 4 \cdot 3 \cdot 2 \cdot 1 = 4!$

$n=4$: $a_5 = 5a_4 = 5(24) = 120 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$

1, 2, 6, 24, 120

FIND $7! = 7 \cdot 6 \cdot \underbrace{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}_{5!} = 7 \cdot 6 \cdot 120$

$= 42 \cdot 120$

$= 5040$